

Book Review: *Ergodic Theorems*

Ergodic Theorems by U. Krengel, with a supplement by A. Brunel de Gruyter, Berlin, 1985. (Studies in Mathematics 6, Vol. VIII, 357 pp.)

Loosely speaking, an ergodic theorem is a statement about the existence of a time average, while ergodicity is the property that a time average is equal to a space average. Time averages are of course time invariant quantities, and ergodicity is also the statement that the time averages are constant on the phase space (or on some subsets). It turns out that up to now most concrete proofs of ergodicity use ergodic theorems at some key steps.

The most important ergodic theorem (known as Birkhoff's ergodic theorem) states that the time average exists almost everywhere with respect to some time invariant phase space average (for example the Liouville measure on a surface of constant energy in classical mechanics). A more recent and also very useful result is Kingman's subadditive ergodic theorem. This powerful theorem is used to prove the Oseledec-Ruelle result about the existence of Lyapunov exponents. These exponents give the exponential growth (decay) rate of an initial disturbance, and are now measured in several hydrodynamics experiments.

The book under review deals mostly with abstract theorems. It contains a fairly complete account of the standard results, but includes also some of the most recent developments (like the above mentioned Oseledec-Ruelle theorem). The book contains also many interesting applications to the theory of stochastic processes. (in particular there is an appendix by D. Brunel on Harris processes.)

This book is well-written and easy to use since chapters are made as independent as possible. The bibliography is excellent and very up-to-date. This is not, however, an introductory book, and the reader is assumed to have a good basic knowledge of the questions addressed by ergodic theory (together with some notions of abstract measure theory). This book will certainly remain for a long a time as a (unique) very useful general reference for this area of mathematics.

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